

Appendix

Foundation Mathematics

Aim: To enable students to develop an adequate foundation upon which to build the additional mathematical skills required for successful actuarial practice.

1 FUNCTIONS AND SETS

- 1.1 Define a function and explain and apply functional concepts including: domain, codomain, image, limit, and inverse.
- 1.1 Determine asymptotes and turning points, and sketch a curve.
- 1.2 Explain basic set terminology and apply basic set concepts.
- 1.3 Define the supremum and infimum of a set of numbers.
- 1.4 Apply simple numerical techniques to calculate roots of equations and evaluate integrals.

2 DIFFERENTIATION

- 2.1 Define the derivative of a function as a limit and determine the derivative from first principles.
- 2.2 Apply the basic rules of differentiation (including the chain rule and implicit differentiation) to calculate first, higher-order, and partial derivatives.
- 2.3 State the derivatives for power, trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic, and inverse hyperbolic functions.
- 2.4 Determine the extreme points of a function of two variables, including using Lagrange multipliers for constrained problems.

3 INTEGRATION

- 3.1 Evaluate definite and indefinite integrals, using basic techniques including substitution and integration by parts.
- 3.2 Evaluate double and triple integrals and calculate areas and volumes of simple geometric shapes.
- 3.3 Interchange the order of integration of multiple integrals and change variables to evaluate multiple integrals.
- 3.4 Apply simple numerical integration techniques such as the trapezium rule and Simpson's rule.

4 SEQUENCES AND SERIES

- 4.1 State the Taylor and Maclaurin expansions for functions of one and two variables.
- 4.2 Define sequence and series and explain the concepts of boundedness, convergence, limit, and monotonicity.
- 4.3 Use the formulae for the sums of arithmetic and geometric progressions.
- 4.4 Use appropriate techniques to determine convergence or boundedness sequences and series in simple cases.

5 DIFFERENTIAL EQUATIONS

- 5.1 Solve first-order differential equations which are separable, linear or homogeneous.
- 5.2 Solve simple first-order differential equation models for various applications with given conditions and use the solution to find the values of any parameters involved.

6 REAL AND COMPLEX NUMBERS

- 6.1 Carry out arithmetic with complex numbers.

7 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

- 7.1 Carry out simple operations with matrices (addition, scalar multiplication, matrix multiplication, transposition).
- 7.2 Calculate the determinant of a matrix and use Cramer's rule to solve a system of linear equations.
- 7.3 Use Gaussian elimination to find the rank of a matrix, to invert a matrix, and to solve systems of linear equations.
- 7.4 Compute the characteristic polynomial of a matrix and determine its eigenvalues and eigenvectors.
- 7.5 Determine whether a given matrix is diagonalizable and, if so, find a diagonalizing matrix.

8 VECTORS, VECTOR SPACES AND INNER PRODUCT SPACES

- 8.1 Carry out simple operations with vectors (addition, scalar product, vector product, scalar triple product).
- 8.2 Explain the concepts of vector space, inner product space, orthogonality.

9 PROBABILITY

- 9.1 Explain what is meant by a set function, a sample space for an experiment, and an event.
- 9.2 Define probability as a set function on a collection of events, stating basic axioms.
- 9.3 Derive basic properties satisfied by the probability of occurrence of an event, and calculate probabilities of events in simple situations.
- 9.4 Derive the addition rule for the probability of the union of two events, and use the rule to calculate probabilities.
- 9.5 Define the conditional probability of one event given the occurrence of another event, and calculate such probabilities.
- 9.6 Derive Bayes' Theorem for events, and use the result to calculate probabilities.
- 9.7 Define correlation and independence for two events, and calculate probabilities in situations involving independence.